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Exact Scalar Field Inflationary Solution in Rainbow Universe

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Abstract Using the Friedmann equation in rainbow Universe, we obtain an exact scalar field Inflationary Solution, which is a modification of the exact scalar field with negative potential $-V_0 + m^2\varphi^2/2$. Because the rainbow metric is Finsler metric, the result in this paper implies that the research of Finsler geometry in Cosmology should lead to several new physics theories.

Keywords Inflationary universe · Scalar field · Rainbow metric · Finsler geometry

1 Introduction

After Gamow's the Big Bang theory of the universe, people have owned several correct predictions from this theory. Now, the Big Bang theory has become the standard cosmological model. However, at the same time, this theory encountered a number of difficulties, such as the famous the monopole problem, the horizon problems and so on. In the early 1980s, Guth et al. put forward the Inflationary universe model to solve the difficulties. Many of inflationary universes model have potential function $V(\varphi)$ with a scalar field φ, where the potential function related to the evolution of the universe, and the researching about negative potential is meaningful in the study of branch cosmology and the accelerate expansion of the universe. However, the equations in the theory is very complex, so, for a long time, people could only use numerical method or approximation methods to get some approximate solution, and the slow-roll approximate, which ignore some slow change terms, could get some meaningful results. However, as well as we known, any weak violation could result in great deviations from the standard predications for the inflation of universe, that is to say that even if the approximate theory is very beautiful, it could fail in some cases, so people want to obtain the exact solutions of the theory. When the equations are resolved, people always assume that the form of the universe potential, and then solve the results of the Hubble factor and scalar field. But if we assume that the probing form of the Hubble factor, and then further

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obtain the potential and scalar field, the difficulty of solving the equations greatly reduce [1–3].

Recently, in quantum gravity research, the rainbow metric is proposed [4–7]. In the theory the nature of space-time correlates with the energy of the probe particle, and when the energy of the probe particles is null, the metric should degenerate as the ordinary metric. Girelli et al. think that the rainbow metric is no other than the Finsler metric [8, 9], so we seem to see that the application of the Finsler geometry in the relativity is very important. The R-W metric, which is the metric of cosmological standard model, can be expressed as the rainbow metric [4, 5], and the modified universe metric can resolve some difficult (for example, horizon problem), and the speed of light and the gravitational factor is no longer a constant. The theory express that the nature of our universe correlate with the energy of probe particles. The interesting conclusion could be observed in the research about the high energy particles. Now, we use the rainbow universe metric to study the exact inflationary universe with a negative potential.

2 Rainbow Universe Metric

When the energy of the probe practical is low, we can consider the Rainbow-Friedmann-Robertson-Walker universe metric [4]

$$ds^2 = -f^{-2}dt^2 + a^2(t)\left[dx^2 + dy^2 + dz^2\right] \tag{1}$$

where f is a energy-depend modification of the metric. Once the energy of the probe particle is vanish, $f = 1$. We can get the non-vanish associated connection read as

$$\Gamma^0_{00} = -\dot{f}/f, \qquad \Gamma^0_{ij} = f^2\dot{a}a\delta_{ij}, \qquad \Gamma^i_{0j} = \delta^i_j\dot{a}/a \tag{2}$$

define

$$\dot{B} = \partial B/\partial t, \qquad B' = \partial B/\partial\varphi \tag{3}$$

and we obtain

$$R_{00} = 3\ddot{a}a^{-1} + 3\dot{a}\dot{f}a^{-1}f^{-1}, \qquad R_{ij} = -(f^2(\ddot{a}a + 2\dot{a}^2) + \dot{f}f\dot{a}a)\delta_{ij} \tag{4}$$

In this paper, we define the scalar filed is $\varphi(t)$, the potential is $V(\varphi)$, the energy density and pressure are $\rho(t)$ and $p(t)$. The conservation equation $T^{ab}_{;b} = 0$ read as

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{5}$$

where

$$H = \frac{\dot{a}}{a} \tag{6}$$

is the Hubble factor, and (5) should be equivalent to the Klein-Gordon equation

$$\ddot{\varphi} + 3H\dot{\varphi} + \dot{f}f^{-1}\dot{\varphi} + V'f^{-2} = 0 \tag{7}$$

since

$$\rho = \frac{1}{2}\dot{\varphi}^2 + U + A \tag{8}$$

$$p = \frac{1}{2}\dot{\varphi}^2 - U - A \tag{9}$$

where

$$V=\int f^2 dU,\qquad A=\int \dot{f} f^{-1}\dot{\varphi}^2 dt \tag{10}$$

From (4) and the Einstein equation

$$H^2=\kappa\rho/3f^2 \tag{11}$$

$$\dot{H}=-\kappa\left(\rho+p\right)/2f^2-H\dot{f}/f \tag{12}$$

where $\kappa=8\pi G$, and the G is Gravitational constant. Using (5)–(9), and transforming (12), we can get

$$2f^2\dot{H}+2H\dot{f}f+3f^2H^2=-\kappa p \tag{13}$$

From (5)–(13), we have

$$\dot{\varphi}=-\frac{2}{\kappa}f\left(fH\right)' \tag{14}$$

$$\begin{aligned}U=&\frac{3}{\kappa}f^2H^2+\frac{2}{\kappa^2}\left[f\left(fH\right)'\right]^2-\frac{4f^3}{\kappa^2}H'\left(fH\right)'-\frac{4Hf^2}{\kappa^2}f'\left(fH\right)'\\&-\frac{4}{\kappa^2}\int \dot{f}f\left[\left(fH\right)'\right]^2dt\end{aligned} \tag{15}$$

Usually, people always assume the potential function V, and then solve other physical quantity. Undoubtedly, the way is very difficult. Now, we can assume the form of the φ-dependent Hubble factor. Next, get the potential and scalar field function from (15), (10) and (14). Finally, we can obtain the form of the scalar factor $a(t)$ from (6). We can get the exact solution easy, and we consider the Hubble factor is

$$H\left(\varphi\right)=bf^{-1}\varphi \tag{16}$$

where b is a constant, and from (16) and (14), we get

$$\varphi=\varphi_0-\frac{2b}{\kappa}\int f dt \tag{17}$$

Similarly, from (10), (16) and (15) we can obtain

$$\begin{aligned}U'&=6b^2\varphi/\kappa-8b^2ff'/\kappa^2\\V&=\frac{1}{2}m^2\int f^2d\varphi^2-V_0f^4\end{aligned} \tag{18}$$

where

$$m^2=\frac{6}{\kappa}b^2,\qquad V_0=\frac{2b^2}{\kappa^2} \tag{19}$$

From (22), we get the time-dependent scalar factor is

$$a=a_0\exp\left(b\varphi_0\int f^{-1}dt-\frac{2b^2}{\kappa}\int f^{-1}\left(\int f dt\right)dt\right) \tag{20}$$

where $a_0=a(t_0)$, and, after the discussion, we observe the solutions of the rainbow universe are no longer an easy form. The universe solutions depend on the energy of probe particles.

3 Discussion

When the energy of probe particles is very litter, the influence of f could be ignored ($f=1$). In fact, (1) is Robertson-Walker metric, and we will emphasize the case in this paper, so we can get the Hubble factor, potential and scalar factor

$$H(\varphi)=b\varphi \tag{21}$$

$$\varphi=\varphi_0-\frac{2b}{\kappa} \tag{22}$$

$$V=-V_0+\frac{1}{2}m^2\varphi^2 \tag{23}$$

$$a=a_0\exp\left(b\varphi_0 t-\frac{b^2}{\kappa}t^2\right) \tag{24}$$

This is a scalar field universe with resonance potential. Of course, if the energy of probe particles can't be ignored, we can get the modification results. For example

$$f=\sqrt{1-l_p^2\bar{\varepsilon}^2(t)} \tag{25}$$

and

$$f=\left(1-l_p^4\bar{\varepsilon}^4(t)\right)^{-1/2} \tag{26}$$

Where l_p is Planck length, so modification term represents the quantum effect, and $\bar{\varepsilon}(t)$ is the average energy of the probe particle when time coordinate is t. We can see that the modification solution will be very perplexing, and it may reveal the fine structure of the universe. In the research of scalar field Inflationary theory, some important parameters read as

$$\varepsilon(\varphi)\equiv\frac{2}{\kappa}\left(\frac{H'(\varphi)}{H(\varphi)}\right)^2 \tag{27}$$

$$\eta(\varphi)\equiv\frac{2}{\kappa}\frac{H''(\varphi)}{H(\varphi)} \tag{28}$$

$$\xi(\varphi)\equiv\frac{2}{\kappa}\left[\frac{H'''(\varphi)H'(\varphi)}{H^2(\varphi)}\right]^{1/2} \tag{29}$$

As well as we know, nowadays, the influence of f should be ignored, so we can think $f=1$. When the universe was inflating, we could not ignore the energy of probe particles, but we can set that the modification from f is very little, so that we still can think $f=1$. If the hypothesis is reasonable, from (27)–(29), we could get $\eta=\xi=0$. So we can let the scalar field at the end of inflation φ_f, and

$$\varepsilon(\varphi_f)=1 \tag{30}$$

From (27) to (30), we have

$$\varphi_f=\sqrt{\frac{2}{\kappa}} \tag{31}$$

If the theory accord with the observation, the universe at the end of inflation should be e^{60} times more than the universe at the beginning of inflation. The number of e-folds of inflation

$$N \equiv \ln \frac{a_e}{a_f} = \int_{\varphi}^{\varphi_f} \frac{H}{\dot{\varphi}} d\varphi = -\frac{\kappa}{2} \int_{\varphi}^{\varphi_f} \frac{H}{H'} d\varphi = \frac{\kappa}{4}(\varphi^2 - \varphi_f^2) \tag{32}$$

We need $N \geq 60$, and

$$N = \frac{\kappa}{4}(\varphi_{60}^2 - \varphi_f^2) = 60 \tag{33}$$

so

$$\varepsilon\,(\varphi_{60}) = \frac{2}{\kappa \varphi_{60}^2} \approx \frac{1}{121} \tag{34}$$

The scalar density fluctuations is given to second order by

$$1 - n_s = 4\varepsilon - 2\eta + 8(1+c)\varepsilon^2 - 2(3+5c)\varepsilon\eta + 2c\varepsilon\xi \tag{35}$$

We can obtain [1]

$$n_s \approx 0.967 \tag{36}$$

Now, the Wilkinson-microwave anisotropy probe (WMAP) data are [10, 11] $n_s = 0.951^{+0.015}_{-0.019}$, and when $f \neq 1$, we deal f with fine modification, and we could get the reasonable n_s.

Recent astronomical observations prove that there are a lot of dark matter and dark energy in our universe, so many scientists believe that current theoretical physics should be modified, then several theories, such as quantum gravity theories and superstring theory, is put forward to explain these problems. However, we believe that a new physical theory may base on a new mathematical theory. Today, the relativity and its conclusions which base on the Riemannian geometry has matured [12–16], but the physics, which base on the more general geometry (Finsler geometry) still has never been study largely. However, because Finsler geometry is differential geometry without the restraint of quadratic form, the geometry is more general than the Riemannian geometry, and the geometry may reveal more of the physical nature. In fact, the form of the modification term in the rainbow universe metric is uncertain. Due to the introduction of modification term, the rainbow metric correlate with energy of the probe particles, and the metric depend tangential vector as well as coordinates in math. Therefore, we think that the modification term of rainbow metric should be determined with the Finsler physics theory. Because of the modification from f, the solutions of the universe have complicated fine structure, which may contain more information about the universe, so the research of Finsler rainbow universe is significant.

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